Chapter 21: Nuclear Chemistry

Unit 134 – Nuclear Structure and Stability

134-1 Write the following isotopes in hyphenated form (e.g., "carbon-14")

(a) $^{24}_{11}Na$

(b) $^{29}_{13}Al$

(c) $^{73}_{36}Kr$

(d) $^{194}_{77}Ir$

Solution

(a) sodium-24; (b) aluminum-29; (c) krypton-73; (d) iridium-194

134-2 Write the following isotopes in nuclide notation (e.g., "$^{14}_{6}C$")

(a) oxygen-14
(b) copper-70
(c) tantalum-175
(d) francium-217

Solution

(a) $^{14}_{8}O$; (b) $^{70}_{29}Cu$; (c) $^{175}_{73}Ta$; (d) $^{217}_{87}Fr$

134-3 For the following isotopes that have missing information, fill in the missing information to complete the notation

(a) $^{34}_{14}X$

(b) $^{36}_{X}P$

(c) $^{57}_{X}Mn$

(d) $^{121}_{56}X$

Solution

(a) $^{34}_{14}Si$; (b) $^{36}_{15}P$; (c) $^{57}_{25}Mn$; (d) $^{121}_{56}Ba$

134-4 For each of the isotopes in Exercise 1, determine the numbers of protons, neutrons, and electrons in a neutral atom of the isotope.

Solution

(a) sodium-24: 11 protons, 13 neutrons, 11 electrons; (b) aluminum-29: 13 protons, 16 neutrons, 13 electrons; (c) krypton-73: 36 protons, 37 neutrons, 36 electrons; (d) iridium-194: 77 protons, 117 neutrons, 77 electrons

134-5 Write the nuclide notation, including charge if applicable, for atoms with the following characteristics:

(a) 25 protons, 20 neutrons, 24 electrons
(b) 45 protons, 24 neutrons, 43 electrons
(c) 53 protons, 89 neutrons, 54 electrons
(d) 97 protons, 146 neutrons, 97 electrons

Solution

(a) ${}^{45}_{25}\text{Mn}^{+1}$; (b) ${}^{69}_{45}\text{Rh}^{+2}$; (c) ${}^{142}_{53}\text{I}^{-1}$; (d) ${}^{243}_{97}\text{Bk}$

134-6 Calculate the density of the ${}^{24}_{12}\text{Mg}$ nucleus in g/mL, assuming that it has the typical nuclear diameter of 1×10^{-13} cm and is spherical in shape.

Solution

$$\frac{24.31\text{ amu} \times 1.6605 \times 10^{-24}\text{ g amu}^{-1}}{\frac{4}{3}(3.1416)(1 \times 10^{-13}\text{ cm}^2)^3} = \frac{4.037 \times 10^{-23}\text{ g}}{5.236 \times 10^{-40}\text{ cm}^3} = 8 \times 10^{16}\text{ g cm}^{-3}$$

134-7 What are the two principal differences between nuclear reactions and ordinary chemical changes?

Solution

Nuclear reactions usually change one type of nucleus into another; chemical changes rearrange atoms. Nuclear reactions involve much larger energies than chemical reactions and have measureable mass changes.

134-8 The mass of the atom ${}^{23}_{11}\text{Na}$ is 22.9898 amu.

(a) Calculate its binding energy per atom in millions of electron volts.
(b) Calculate its binding energy per nucleon.

Solution

(a) The binding energy per atom is calculated from the mass defect, the difference between the actual mass of the nuclide and its component parts. First, determine the theoretical mass of ${}^{23}_{11}\text{Na}$, which contains 11 protons, 11 electrons, and 12 neutrons:

protons: 11×1.0073 amu = 11.0803 amu

electrons: 11×0.00055 amu = 0.00605 amu

neutrons: 11×1.0087 amu = <u>12.1044 amu</u>

theoretical mass: 23.1908 amu

mass defect = 23.1908 amu – 22.9898 amu = 0.2010 amu

To use the Einstein conversion, the mass must be expressed in kilograms:

$$\text{mass defect} = 0.2010\text{ amu} \times \frac{1.6605 \times 10^{-27}\text{ kg}}{1\text{ amu}} = 3.3376 \times 10^{-28}\text{ kg}$$

$E = mc^2 = (3.3376 \times 10^{-28}\text{ kg})(2.9979 \times 10^8\text{ m/s})^2$

$= 2.9996 \times 10^{-11}\text{ kg m}^2/\text{s}^2 = 2.9996 \times 10^{-11}$ J per nucleon;

In terms of megaelectron volts, use the conversion factor:

1 MeV = 1.602189×10^{-13} J, which gives:

2.996×10^{-11} J/nucleus $\times \dfrac{1\ \text{MeV}}{1.602189 \times 10^{-13}\ \text{J}}$ = 187.2 MeV per nucleus;

(b) 187.2 MeV/nucleus $\times \dfrac{1\ \text{nucleus}}{23\ \text{nucleons}}$ = 8.140 MeV per nucleon]

134-9 Which of the following nuclei lie within the band of stability shown in Figure 21.2?

(a) chlorine-37
(b) calcium-40
(c) ^{204}Bi
(d) ^{56}Fe
(e) ^{206}Pb
(f) ^{211}Pb
(g) ^{222}Rn
(h) carbon-14

Solution

(a), (b), (c), (d), and (e)

134-10 Which of the following nuclei lie within the band of stability shown in Figure 21.2?

(a) argon-40
(b) oxygen-16
(c) ^{122}Ba
(d) ^{58}Ni
(e) ^{205}Tl
(f) ^{210}Tl
(g) ^{226}Ra
(h) magnesium-24

Solution

(a), (b), (d), (e), and (h)

Unit 135 – Nuclear Equations

135-1 Write a brief description or definition of each of the following:

(a) nucleon
(b) α particle
(c) β particle
(d) positron
(e) γ ray
(f) nuclide
(g) mass number
(h) atomic number

Solution

(a) A nucleon is any particle contained in the nucleus of the atom, so it can refer to protons and neutrons. (b) An α particle is one product of natural radioactivity and is the nucleus of a helium atom. (c) A β particle is a product of natural radioactivity and is a high-speed electron. (d) A positron is a particle with the same mass as an electron but with a positive charge. (e) Gamma rays compose electromagnetic radiation of high energy and short wavelength. (f) Nuclide is a term used when referring to a single type of nucleus. (g) The mass number is the sum of the number of protons and the number of neutrons in an element. (h) The atomic number is the number of protons in the nucleus of an element.

135-2 Which of the various particles (α particles, β particles, and so on) that may be produced in a nuclear reaction are actually nuclei?
Solution
α particles are He-4 nuclei; protons are H-1 nuclei.

135-3 Complete each of the following equations by adding the missing species:

(a) ${}^{27}_{13}\text{Al} + {}^{4}_{2}\text{He} \longrightarrow ? + {}^{1}_{0}\text{n}$

(b) ${}^{239}_{94}\text{Pu} + ? \longrightarrow {}^{242}_{96}\text{Cm} + {}^{1}_{0}\text{n}$

(c) ${}^{235}_{92}\text{U} \longrightarrow ? + {}^{135}_{55}\text{Cs} + 4{}^{1}_{0}\text{n}$

(d) ${}^{235}_{92}\text{U} \longrightarrow ? + {}^{135}_{55}\text{Cs} + 4{}^{1}_{0}\text{n}$

Solution
(a) ${}^{27}_{13}\text{Al} + {}^{4}_{2}\text{He} \longrightarrow {}^{30}_{15}\text{P} + {}^{1}_{0}\text{n}$; (b) ${}^{239}_{94}\text{Pu} + {}^{4}_{2}\text{He} \longrightarrow {}^{242}_{96}\text{Cm} + {}^{1}_{0}\text{n}$; (c) ${}^{14}_{7}\text{N} + {}^{4}_{2}\text{He} \longrightarrow {}^{17}_{8}\text{O} + {}^{1}_{1}\text{H}$; (d) ${}^{235}_{92}\text{U} \longrightarrow {}^{96}_{37}\text{Rb} + {}^{135}_{55}\text{Cs} + 4{}^{1}_{0}\text{n}$

135-4 Complete each of the following equations:

(a) ${}^{7}_{3}\text{Li} + ? \longrightarrow 2{}^{4}_{2}\text{He}$

(b) ${}^{14}_{6}\text{C} \longrightarrow {}^{14}_{7}\text{N} + ?$

(c) ${}^{27}_{13}\text{Al} + {}^{4}_{2}\text{He} \longrightarrow ? + {}^{1}_{0}\text{n}$

(d) ${}^{250}_{96}\text{Cm} \longrightarrow ? + {}^{98}_{38}\text{Sr} + 4{}^{1}_{0}\text{n}$

Solution
(a) ${}^{7}_{3}\text{Li} + {}^{1}_{1}\text{p} \longrightarrow 2{}^{4}_{2}\text{He}$; (b) ${}^{14}_{6}\text{C} \longrightarrow {}^{14}_{7}\text{N} + {}^{0}_{-1}\text{e}$; (c) ${}^{27}_{13}\text{Al} + {}^{4}_{2}\text{He} \longrightarrow {}^{30}_{15}\text{P} + {}^{1}_{0}\text{n}$; (d) ${}^{250}_{96}\text{Cm} \longrightarrow {}^{148}_{58}\text{Ce} + {}^{98}_{38}\text{Sr} + 4{}^{1}_{0}\text{n}$

135-5 Write a balanced equation for each of the following nuclear reactions:

(a) the production of ${}^{17}\text{O}$ from ${}^{14}\text{N}$ by α particle bombardment
(b) the production of ${}^{14}\text{C}$ from ${}^{14}\text{N}$ by neutron bombardment
(c) the production of ${}^{233}\text{Th}$ from ${}^{232}\text{Th}$ by neutron bombardment
(d) the production of ${}^{239}\text{U}$ from ${}^{238}\text{U}$ by ${}^{2}_{1}\text{H}$ bombardment

Solution
(a) ${}^{14}_{7}\text{N} + {}^{4}_{2}\text{He} \longrightarrow {}^{17}_{8}\text{O} + {}^{1}_{1}\text{H}$; (b) ${}^{14}_{7}\text{C} + {}^{1}_{0}\text{n} \longrightarrow {}^{14}_{6}\text{C} + {}^{1}_{1}\text{H}$; (c) ${}^{232}_{90}\text{Th} + {}^{1}_{0}\text{n} \longrightarrow {}^{233}_{90}\text{Th}$;
(d) ${}^{238}_{92}\text{U} + {}^{2}_{1}\text{H} \longrightarrow {}^{239}_{92}\text{U} + {}^{1}_{1}\text{H}$

135-6 Technetium-99 is prepared from ^{98}Mo. Molybdenum-98 combines with a neutron to give molybdenum-99, an unstable isotope that emits a β particle to yield an excited form of technetium-99, represented as $^{99}Tc^*$. This excited nucleus relaxes to the ground state, represented as ^{99}Tc, by emitting a γ ray. The ground state of ^{99}Tc then emits a β particle. Write the equations for each of these nuclear reactions.

Solution

$^{98}_{42}Mo + ^{1}_{0}n \longrightarrow ^{99}_{42}Mo$

$^{99}_{42}Mo \longrightarrow ^{99}_{43}Tc^* + ^{0}_{-1}e$

$^{99}_{43}Tc^* \longrightarrow ^{99}_{43}Tc + \gamma$

$^{99}_{43}TC \longrightarrow ^{99}_{44}Ru + ^{0}_{-1}e$]

135-7 The mass of the atom $^{19}_{9}F$ is 18.99840 amu.

(a) Calculate its binding energy per atom in millions of electron volts.
(b) Calculate its binding energy per nucleon.

Solution

(a) Determine the mass defect of the nuclide, which is the difference between the mass of 9 protons, 10 neutrons, and 9 electrons, and the observed mass of a $^{19}_{9}F$ atom:

mass defect = [(9 × 1.0073 amu) + (10 × 1.0087 amu) + (9 × 0.00055 amu)] – 18.99840 amu = 19.15765 amu – 18.99840 amu = 0.15925 amu;

$$E = mc^2 = 0.15925 \text{ amu} \times \frac{1.6605 \times 10^{-27} \text{ kg}}{1 \text{ amu}} \times \left(2.998 \times 10^8 \frac{m}{s}\right)^2$$

$= 2.377 \times 10^{-11}$ kg m/s^2
$= 2.377 \times 10^{-11}$ J

$$2.337 \times 10^{-11} \text{ J} \times \frac{1 \text{ MeV}}{1.602 \times 10^{-13} \text{ J}} = 148.8 \text{ MeV per atom ;}$$

(b) Binding energy per nucleon = $\frac{148.4 \text{ MeV}}{19}$ = 7.808 MeV/nucleon

135-8 For the reaction $^{14}_{6}C \longrightarrow ^{14}_{7}N + ?$, if 100.0 g of carbon reacts, what volume of nitrogen gas (N_2) is produced at 273K and 1 atm?

Solution

$$100.0 \text{ g } ^{14}C \times \frac{1 \text{ mol } ^{14}C}{14 \text{ g } ^{14}C} \times \frac{1 \text{ mol } ^{14}N}{1 \text{ mol } ^{14}C} \times \frac{1 \text{ mol } ^{14}N_2}{2 \text{ mol } ^{14}N} = 3.57 \text{ mol } N_2$$

$$V = \frac{nRT}{P} = \frac{(3.57 \text{ mol } N_2)(0.0821 \text{ L atm/mol K})(273 \text{ K})}{1 \text{ atm}} = 80.0 \text{ L}$$

Unit 136 – Radioactive Decay

136-1 What are the types of radiation emitted by the nuclei of radioactive elements?
Solution
α (helium nuclei), β (electrons), β^+ (positrons), and η (neutrons) may be emitted from a radioactive element, all of which are particles; γ rays also may be emitted.

136-2 What changes occur to the atomic number and mass of a nucleus during each of the following decay scenarios?
(a) an α particle is emitted
(b) a β particle is emitted
(c) γ radiation is emitted
(d) a positron is emitted
(e) an electron is captured
Solution
(a) The atomic number decreases by two and the mass number decreases by four. (b) The atomic number increases by one and the mass number remains unchanged. (c) The atomic number and mass number remain unchanged. (d) The atomic number decreases by one and mass number remains unchanged. (e) The atomic number decreases by one and the mass number remains unchanged.

136-3 What is the change in the nucleus that results from the following decay scenarios?
(a) emission of a β particle
(b) emission of a β^+ particle
(c) capture of an electron
Solution
(a) conversion of a neutron to a proton: ${}^{1}_{0}\text{n} \longrightarrow {}^{1}_{1}\text{p} + {}^{0}_{+1}\text{e}$; (b) conversion of a proton to a neutron; the positron has the same mass as an electron and the same magnitude of positive charge as the electron has negative charge; when the n:p ratio of a nucleus is too low, a proton is converted into a neutron with the emission of a positron: ${}^{1}_{1}\text{p} \longrightarrow {}^{1}_{0}\text{n} + {}^{0}_{+1}\text{e}$; (c) In a proton-rich nucleus, an inner atomic electron can be absorbed. In simplifies form, this changes a proton into a neutron: ${}^{1}_{1}\text{p} + {}^{0}_{-1}\text{e} \longrightarrow {}^{1}_{0}\text{n}$

136-4 Many nuclides with atomic numbers greater than 83 decay by processes such as electron emission. Explain the observation that the emissions from these unstable nuclides also normally include α particles.
Solution
Large isotopes tend to undergo decay, emitting α particles and sometimes large masses to decrease the n:p ratio faster.

136-5 Why is electron capture accompanied by the emission of an X-ray?
Solution

The electron pulled into the nucleus was most likely found in the 1*s* orbital. As an electron falls from a higher energy level to replace it, the difference in the energy of the replacement electron in its two energy levels is given off as an X-ray.

136-6 Explain, in terms of Figure 21.2, how unstable heavy nuclides (atomic number > 83) may decompose to form nuclides of greater stability (a) if they are below the band of stability and (b) if they are above the band of stability.

Solution

(a) For nuclides below the band of stability, the number of neutrons is too small compared with the number of protons. Alpha decay for heavy elements will increase the n:p ratio. Positron (β^+) emission or electron capture will increase the number of neutrons and hence increase the n:p ratio. (b) For nuclides above the band of stability, β^- (electron) emission increases the number of protons and hence decreases the n:p ratio.

136-7 Which of the following nuclei is most likely to decay by positron emission? Explain your choice.

(a) chromium-53
(b) manganese-51
(c) iron-59

Solution

Manganese-51 is most likely to decay by positron emission. The n:p ratio for Cr-53 is $\frac{29}{24}$ = 1.21; for Mn-51, it is $\frac{26}{25}$ = 1.04; for Fe-59, it is $\frac{33}{26}$ = 1.27. Positron decay occurs when the n:p ratio is low. Mn-51 has the lowest n:p ration and therefore is most likely to decay by positron emission. Besides, ${}^{53}_{24}\text{Cr}$ is a stable isotope, and ${}^{59}_{26}\text{Fe}$ decays by beta emission.

136-8 The following nuclei do not lie in the band of stability. How would they be expected to decay? Explain your answer.

(a) ${}^{34}_{15}\text{P}$
(b) ${}^{239}_{92}\text{U}$
(c) ${}^{38}_{20}\text{Ca}$
(d) ${}^{3}_{1}\text{H}$
(e) ${}^{245}_{94}\text{Pu}$

Solution

(a) too many neutrons, β decay; (b) atomic number greater than 82, α or β decay; (c) too few neutrons, electron capture or positron emission; (d) too many neutrons, β decay; (e) atomic number greater than 82, α or β decay

136-9 The following nuclei do not lie in the band of stability. How would they be expected to decay?

(a) ${}^{28}_{15}\text{P}$
(b) ${}^{235}_{92}\text{U}$
(c) ${}^{37}_{20}\text{Ca}$
(d) ${}^{9}_{3}\text{Li}$

(e) ${}^{245}_{96}\text{Cm}$

Solution

(a) too many neutrons, β^+ decay; (b) atomic number greater than 82, α decay; (c) too few neutrons, positron emission; (d) too many neutrons, β decay; (e) atomic number greater than 83, α decay

136-10 Predict by what mode(s) of spontaneous radioactive decay each of the following unstable isotopes might proceed:

(a) ${}^{6}_{2}\text{He}$

(b) ${}^{60}_{30}\text{Zn}$

(c) ${}^{235}_{91}\text{Pa}$

(d) ${}^{241}_{94}\text{Np}$

(e) ${}^{18}\text{F}$

(f) ${}^{129}\text{Ba}$

(g) ${}^{237}\text{Pu}$

Solution

(a) n:p ratio high, β decay; (b) n:p ratio low, positron emission or electron capture; (c) n:p ratio high and Z > 82, α or β decay; (d) n:p ratio high, α or β decay; (e) n:p ratio low, positron emission or electron capture; (f) n:p ratio low, positron emission or electron capture; (g) n:p ratio low, positron emission or electron capture

136-11 Write a nuclear reaction for each step in the formation of ${}^{218}_{84}\text{Po}$ from ${}^{238}_{98}\text{U}$, which proceeds by a series of decay reactions involving the step-wise emission of α, β, β, α, α, α particles, in that order.

Solution

$${}^{238}_{92}\text{U} \longrightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He};\ {}^{234}_{90}\text{Th} \longrightarrow {}^{234}_{91}\text{Pa} + {}^{0}_{-1}\text{e};\ {}^{234}_{91}\text{Pa} \longrightarrow {}^{234}_{92}\text{U} + {}^{0}_{-1}\text{e};$$
$${}^{234}_{92}\text{U} \longrightarrow {}^{230}_{90}\text{Th} + {}^{4}_{2}\text{He};\ {}^{230}_{90}\text{Th} \longrightarrow {}^{226}_{88}\text{Ra} + {}^{4}_{2}\text{He};\ {}^{226}_{88}\text{Ra} \longrightarrow {}^{222}_{86}\text{Rn} + {}^{4}_{2}\text{He};$$
$${}^{222}_{86}\text{Rn} \longrightarrow {}^{218}_{84}\text{Po} + {}^{4}_{2}\text{He}$$

136-12 Write a nuclear reaction for each step in the formation of ${}^{208}_{82}\text{Pb}$ from ${}^{228}_{90}\text{Th}$, which proceeds by a series of decay reactions involving the step-wise emission of α, α, α, α, β, β, α particles, in that order.

Solution

$${}^{4}_{2}\text{He} = \alpha,\ {}^{0}_{-1}\text{e} = \beta$$
$${}^{228}_{90}\text{Th} \longrightarrow {}^{224}_{88}\text{Ra} + \alpha;\ {}^{224}_{88}\text{Ra} \longrightarrow {}^{220}_{86}\text{Rn} + \alpha;\ {}^{220}_{86}\text{Rn} \longrightarrow {}^{216}_{84}\text{Po} + \alpha;\ {}^{216}_{84}\text{Po} \longrightarrow {}^{212}_{82}\text{Pb} + \alpha;$$
$${}^{212}_{82}\text{Pb} \longrightarrow {}^{212}_{83}\text{Bi} + \beta;\ {}^{212}_{83}\text{Bi} \longrightarrow {}^{212}_{84}\text{Po} + \beta;\ {}^{212}_{84}\text{Po} \longrightarrow {}^{208}_{82}\text{Pb} + \alpha$$

136-13 Define the term half-life and illustrate it with an example.

Solution

Half-life is the time required for half the atoms in a sample to decay. Example (answers may vary): For C-14, the half-life is 5770 years. A 10-g sample of C-14 would contain 5 g of C-14 after 5770 years; a 0.20-g sample of C-14would contain 0.10 g after 5770 years.

136-14 A 1.00×10^{-6}-g sample of nobelium, $^{254}_{102}\text{No}$, has a half-life of 55 seconds after it is formed. What is the percentage of $^{254}_{102}\text{No}$ remaining at the following times?

(a) 5.0 min after it forms
(b) 1.0 h after it forms

Solution

(a) 5.0 minutes is 5.5 half-lives. The fraction that remains after 5.5 half-lives is $\left(\frac{1}{2}\right)^{5.5} = 0.022$ or 2.2%. (b) 1 hour is 65 half-lives. The fraction that remains after 65 half-lives is

$\left(\frac{1}{2}\right)^{65} = 2.7 \times 10^{-10}$ or 2.7×10^{-8}%.

Question 15.

136-15 ^{239}Pu is a nuclear waste byproduct with a half-life of 24,000 y. What fraction of the ^{239}Pu present today will be present in 1000 y?

Solution

1000 years is 0.04 half-lives. The fraction that remains after 0.04 half-lives is $\left(\frac{1}{2}\right)^{0.04} = 0.973$ or 97.3%

136-16 Fluorine-18 is a radioactive isotope that decays by positron emission to form oxygen-18 with a half–life of 109.7 min. (A positron is a particle with the mass of an electron and a single unit of positive charge; the equation is $^{18}_{9}\text{F} \longrightarrow {}^{8}_{18}\text{O} + {}^{0}_{+1}\text{e}$.) Physicians use ^{18}F to study the brain by injecting a quantity of fluoro-substituted glucose into the blood of a patient. The glucose accumulates in the regions where the brain is active and needs nourishment.

(a) What is the rate constant for the decomposition of fluorine-18?
(b) If a sample of glucose containing radioactive fluorine-18 is injected into the blood, what percent of the radioactivity will remain after 5.59 h?
(c) How long does it take for 99.99% of the ^{18}F to decay?

Solution

(a) This is a first-order reaction:

$$t_{1/2} = \frac{\ln 2}{k}$$

$$k = \frac{0.6932}{109.7 \text{ min}} = 6.319 \times 10^{-3} \text{ min}^{-1}$$

(b) $\ln\left(\frac{[A]_0}{[A]}\right) = kt$; let $[A]_0 = 1.000$ (this is 100%):

$$\ln\left(\frac{1}{[A]}\right) = 6.319 \times 10^{-3} \text{ min}^{-1} \times 5.59 \text{ h} \times 60 \text{ min h}^{-1} = 2.119$$

Convert 2.119, a natural log, by taking the *e*s of both sides:

$\frac{1}{[A]} = 8.323$

$[A] = 0.120$

Thus, 12.0% of the radioactivity remains.

(c)

$$\ln \frac{[A]_0}{0.0001\,[A]_0} = 6.319 \times 10^{-3}\ \text{min}^{-1} \times t$$

$$9.210 = 6.319 \times 10^{-3}\ \text{min}^{-1} \times t$$

$$t = 1458\ \text{min}\ (24.29\ \text{h})$$

136-17 The isotope ^{208}Tl undergoes β decay with a half-life of 3.1 min.

(a) What isotope is produced by the decay?

(b) How long will it take for 99.0% of a sample of pure ^{208}Tl to decay?

(c) What percentage of a sample of pure ^{208}Tl remains un-decayed after 1.0 h?

Solution

(a) $^{208}_{81}\text{Ti} \longrightarrow\ ^{0}_{-1}\text{e} +\ ^{208}_{82}\text{Pb}$;

(b) Determine the decay constant λ; then use the integrated rate equation:

$$\lambda = \frac{0.693}{t_{\frac{1}{2}}} = \frac{0.693}{3.1\ \text{min}} = 0.224\ \text{min}^{-1}$$

The percentage of sample remaining after a period of time is independent of the mass. If 99% of the sample has decayed, 1% of the original mass of 100% remains:

$$\ln \frac{c_0}{c} = \lambda t \qquad \ln\frac{100\%}{1\%} = 0.224\ \text{min}^{-1} t$$

$4.605 = (0.224\ \text{min}^{-1})t$

$t = 21$ min;

(c) 1 houris 19.3 half-lives. The fraction thatremainsafter 19.3 half-lives is $\left(\frac{1}{2}\right)^{19.3} = 1.5 \times 10^{-5}$

or $1.5 \times 10^{-4}\%$

136-18 If 1.000 g of $^{226}_{88}\text{Ra}$ produces 0.0001 mL of the gas $^{222}_{86}\text{Rn}$ at STP (standard temperature and pressure) in 24 h, what is the half-life of ^{226}Ra in years?

Solution

$PV = nRT$

$$n_{\text{Rn}} = \frac{PV}{RT} = \frac{(1\ \text{atm})(0.0001\ \text{mL} \times 1\ \text{L}/10^3\ \text{mL})}{(0.08206\ \text{L atm mol}^{-1}\ \text{K}^{-1})(273.15\ \text{K})} = 4.4614 \times 10^{-9}\text{mol}$$

n_{Rn} produced = n_{Rn} decayed

$$\text{mass Ra lost} = 4.4614 \times 10^{-9}\ \text{mol} \times \frac{226\ \text{g}}{\text{mol}} = 1.00827 \times 10^{-6}\ \text{g}$$

mass Ra remaining after 24 h = $1 - (1.00827 \times 10^{-6}\ \text{g}) = 9.9999899 \times 10^{-1}$ g

$$\ln\frac{c_0}{c} = \lambda t = \ln\frac{1.000}{9.9999899 \times 10^{-1}} = \lambda(24\text{ h}) = 4.3785 \times 10^{-7}$$

$$\lambda = 4.2015 \times 10^{-8}\text{ h}^{-1}$$

$$t_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{4.2015 \times 10^{-8}} = 1.6494 \times 10^{7}\text{ h}$$

$$= 1.6494 \times 10^{7}\text{ h} \times \frac{1\text{ d}}{24\text{ h}} \times \frac{1\text{ y}}{365\text{ d}} = 1.883 \times 10^{3}\text{ y or } 2 \times 10^{3}\text{ y}$$

136-19 The isotope ${}^{90}_{38}\text{Sr}$ is one of the extremely hazardous species in the residues from nuclear power generation. The strontium in a 0.500-g sample diminishes to 0.393 g in 10.0 y. Calculate the half-life.

Solution

(This exercise is done using common logs.)

$$\ln\frac{c_0}{c_t} = \lambda t$$

$$\ln\frac{0.500\text{ g}}{0.393\text{ g}} = \lambda(10\text{ y})$$

$$\ln 1.2723 = 10\text{ y}(\lambda)$$

$$\lambda = \frac{0.2408}{10\text{ y}} = 0.02408\text{ y}^{-1}$$

$$\text{Half-life} = \frac{0.693}{0.02408\text{ y}^{-1}} = 28.8\text{ y}$$

136-20 Technetium-99 is often used for assessing heart, liver, and lung damage because certain technetium compounds are absorbed by damaged tissues. It has a half-life of 6.0 h. Calculate the rate constant for the decay of ${}^{99}_{43}\text{Tc}$.

Solution

(Recall that radioactive decay is a first-order process.)

$$\lambda = \frac{0.693}{t_{\frac{1}{2}}} = \frac{0.693}{6.0\text{ h}} = 0.12\text{ h}^{-1}$$

136-21 What is the age of mummified primate skin that contains 8.25% of the original quantity of ${}^{14}\text{C}$?

Solution

Carbon-14 has a half-life of 5730 y. Determine the decay constant and then the age of the skin:

$$\ln\left(\frac{1.00}{0.500}\right) = \lambda t_{1/2}$$

$$\lambda = \frac{\ln\ 2}{t_{1/2}} = \frac{0.6931}{5730\ \text{y}} = 1.21 \times 10^{-4}\ \text{y}^{-1}$$

$$\ln\left(\frac{1.00}{0.0825}\right) = (1.21 \times 10^{-4}\ \text{y}^{-1})t$$

$$t = \frac{2.495}{1.21} \times 10^{-4}\ \text{y}^{-1} = 2.06 \times 10^{4}\ \text{y}$$

136-22 A sample of rock was found to contain 8.23 mg of rubidium-87 and 0.47 mg of strontium-87.
(a) Calculate the age of the rock if the half-life of the decay of rubidium by β emission is 4.7×10^{10} y.
(b) If some $^{87}_{38}\text{Sr}$ was initially present in the rock, would the rock be younger, older, or the same age as the age calculated in (a)? Explain your answer.

Solution

(a) $^{87}_{37}\text{Rb} \longrightarrow\ ^{87}_{38}\text{Sr} +\ ^{0}_{-1}\text{e}$

$^{87}_{38}\text{Sr}$ is a stable isotope and does not decay further. Calculate the value of the decay rate constant for $^{87}_{37}\text{Rb}$, remembering that all radioactive decay is first order:

$$\lambda = \frac{0.693}{4.7 \times 10^{10}\ \text{y}} = 1.47 \times 10^{-11}\ \text{y}^{-1}$$

Calculate the number of moles of $^{87}_{37}\text{Rb}$ and $^{87}_{38}\text{Sr}$ found in the sample at time t:

$$8.23\ \text{mg} \times \frac{1\ \text{g}}{1000\ \text{mg}} \times \frac{1\ \text{mol}}{87.0\ \text{g}} = 9.46 \times 10^{-5}\ \text{mol of}\ ^{87}_{37}\text{Rb}$$

$$0.47\ \text{mg} \times \frac{1\ \text{g}}{1000\ \text{mg}} \times \frac{1\ \text{mol}}{87.0\ \text{g}} = 5.40 \times 10^{-6}\ \text{mol of}\ ^{87}_{38}\text{Sr}$$

Each mol of $^{87}_{37}\text{Rb}$ that disappeared (by radioactive decay of the $^{87}_{37}\text{Rb}$ initially present in the rock) produced 1 mol of $^{87}_{38}\text{Sr}$. Hence the number of moles of $^{87}_{38}\text{Rb}$ that disappeared by radioactive decay equals the number of moles of $^{87}_{38}\text{Sr}$ that were produced. This amount consists of the 5.40×10^{-6}mol of $^{87}_{38}\text{Sr}$ found in the rock at time t if all the $^{87}_{38}\text{Sr}$ present at time t resulted from radioactive decay of $^{87}_{37}\text{Rb}$ and no strontium-87 was present initially in the rock. Using this assumption, we can calculate the total number of moles of rubidium-87 initially present in the rock:

Total number of moles of $^{87}_{37}\text{Rb}$ initially present in the rock at time t_0 = number of moles of $^{87}_{37}\text{Rb}$ at time t + number of moles of $^{87}_{37}\text{Rb}$ that decayed during the time interval $t - t_0$ = number of moles of $^{87}_{37}\text{Rb}$ measured at time t + number of moles of $^{87}_{38}\text{Sr}$ measured at time t = 9.46×10^{-5}mol + 5.40×10^{-6}mol = 1.00×10^{-4}mol

The number of moles can be substituted for concentrations in the expression:

$$\ln\frac{c_0}{c_t} = \lambda t$$

Thus:

$$\ln\frac{1.00 \times 10^{-4}\ \text{mol}}{9.46 \times 10^{-5}\ \text{mol}} = (1.47 \times 10^{-11})t$$

$$t = \left(\ln\frac{1.00 \times 10^{-4}}{9.46 \times 10^{-5}}\right)\left(\frac{1}{1.47 \times 10^{-11}\ \text{y}^{-1}}\right)$$

= 3.8 × 109 y = 3.8 billion years = age of the rock sample;

(b) The rock would be younger than the age calculated in part (a). If Sr was originally in the rock, the amount produced by radioactive decay would equal the present amount minus the initial amount. As this amount would be smaller than the amount used to calculate the age of the rock and the age is proportional to the amount of Sr, the rock would be younger.

136-23 A laboratory investigation shows that a sample of uranium ore contains 5.37 mg of ${}^{238}_{92}\text{U}$ and 2.52 mg of ${}^{206}_{82}\text{Pb}$. Calculate the age of the ore. The half-life of ${}^{238}_{92}\text{U}$ is 4.5×10^9yr.

Solution

Uranium-238 decays into lead-206. First, calculate the decay rate constant for ${}^{238}_{92}\text{U}$:

$$\lambda = \frac{0.693}{4.5 \times 10^9\ \text{y}^{-1}} = 1.54 \times 10^{-10}\ \text{y}^{-1}$$

Calculate the number of moles of ${}^{238}_{92}\text{U}$ and ${}^{206}_{82}\text{Pb}$ found in the sample at time *t*:

$$5.37\ \text{mg} \times \frac{1\ \text{g}}{1000\ \text{mg}} \times \frac{1\ \text{mol}}{238\ \text{g}} = 2.26 \times 10^{-5}\ \text{mol}\ {}^{238}_{92}\text{U}$$

$$2.52\ \text{mg} \times \frac{1\ \text{g}}{1000\ \text{mg}} \times \frac{1\ \text{mol}}{206\ \text{g}} = 1.22 \times 10^{-5}\ \text{mol}\ {}^{206}_{82}\text{Pb}$$

Each mole of ${}^{238}_{92}\text{U}$ that disappeared by radioactive decay produced 1 mol of ${}^{206}_{82}\text{Pb}$. Thus the number of moles of ${}^{206}_{82}\text{Pb}$ produced is equal to the number of ${}^{238}_{92}\text{U}$ that decayed. This amount is the 1.22×10^{-5} mol ${}^{206}_{82}\text{Pb}$ found in the ore at time *t* if all the ${}^{206}_{82}\text{Pb}$ present at time *t* resulted from radioactive decay of ${}^{238}_{92}\text{U}$ and no ${}^{206}_{82}\text{Pb}$ was originally present in the ore.

The total number of moles of ${}^{238}_{92}\text{U}$ initially present in the rock at time t_0 equals the number of moles of ${}^{238}_{92}\text{U}$ measured at time *t* plus the number of moles of ${}^{238}_{92}\text{U}$ that disappeared by radioactive decay during the time interval $t - t_0$. The number of moles of ${}^{238}_{92}\text{U}$ measured at time *t* equals the number of moles of ${}^{206}_{82}\text{Pb}$ measured at time *t*. Therefore:

$$\text{mol}\ {}^{238}_{92}\text{U} = 2.26 \times 10^{-5}\text{mol} + 1.22 \times 10^{-5}\text{mol} = 3.48 \times 10^{-5}\text{mol}$$

Because the volume of the ore sample remains constant, the number of moles can be substituted for concentrations in the expression:

$$\ln\frac{c_0}{c_t} = \lambda t$$

$\ln \frac{3.48 \times 10^{-5}\ \text{mol}}{2.26 \times 10^{-5}\ \text{mol}} = 1.54 \times 10^{-10}\ \text{y}$

$t = (\ln 1.5398) \frac{1}{1.54 \times 10^{-10}\ \text{y}}$

$= (0.4316) \frac{1}{1.54 \times 10^{-10}\ \text{y}}$

$= 2.8 \times 10^{9}\ \text{y}$

136-24 Plutonium was detected in trace amounts in natural uranium deposits by Glenn Seaborg and his associates in 1941. They proposed that the source of this ^{239}Pu was the capture of neutrons by ^{238}U nuclei. Why is this plutonium not likely to have been trapped at the time the solar system formed 4.7×10^{9} years ago?

Solution

$^{239}_{94}\text{Pu}$ has a half-life of 2.411×10^{4} y. Calculate the value of λ and then determine the amount of plutonium-239 remaining after 4.7×10^{9} y:

$\lambda t = \lambda(2.411 \times 10^{4}\ \text{y}) = \ln\left(\frac{1.0000}{0.5000}\right) = 0.6931$

$\lambda = \frac{0.6931}{2.411} \times 10^{4}\ \text{y} = 2.875 \times 10^{-5}\ \text{y}^{-1}$

Then:

$\ln \frac{c_0}{c} = \lambda t$

$\ln\left(\frac{1.000}{c}\right) = 2.875 \times 10^{-5}\ \text{y}^{-1} \times 4.7 \times 10^{9}\ \text{y}$

$\ln c = -1.351 \times 10^{5}$

$c = 0$

This calculation shows that no Pu-239 could remain since the formation of the earth. Consequently, the plutonium now present could not have been formed with the uranium.

136-25 A $^{7}_{4}\text{Be}$ atom (mass = 7.0169 amu) decays into a $^{7}_{3}\text{Li}$ atom (mass = 7.0160 amu) by electron capture. How much energy (in millions of electron volts, MeV) is produced by this reaction?

Solution

The reaction is:

$^{7}_{4}\text{Be} \longrightarrow {}^{7}_{3}\text{Li}$ (electron capture)

7.0169 amu 7.0160 amu

The mass defect is:

$7.0169 - 7.0160 = 9 \times 10^{-4}\text{amu}$

$= 9 \times 10^{-4}\text{amu} \times 1.6605 \times 10^{-27}\ \text{kg/amu}$

$= 1.49 \times 10^{-30}\ \text{kg}$

The energy produced by the reaction is:

$E = mc^2 = (1.49 \times 10^{-30}\ \text{kg})(2.9979 \times 10^{8}\ \text{m s}^{-1})^2$

$= 1.34 \times 10^{-13}\ \text{kg m}^2/\text{s}^2 = 1.34 \times 10^{-13}\ \text{J/nucleus}$

$= 1.34 \times 10^{-13}\ \text{J} \times \dfrac{1\ \text{MeV}}{1.602177 \times 10^{-13}\ \text{J}} = 0.8\ \text{MeV}$

136-26 A $^{8}_{5}\text{B}$ atom (mass = 8.0246 amu) decays into a $^{8}_{4}\text{Be}$ atom (mass = 8.0053 amu) by loss of a β^+ particle (mass = 0.00055 amu) or by electron capture. How much energy (in millions of electron volts) is produced by this reaction?

Solution

Find the mass difference of the starting mass and the total masses of the final products. Then use the conversion for mass to energy to find the energy released:

8.0246 – 8.0053 – 0.00055 = 0.01875 amu

$0.01875\ \text{amu} \times 1.6605 \times 10^{-27}\ \text{kg/amu} = 3.113 \times 10^{-29}\ \text{kg}$

$E = mc^2 = (3.113 \times 10^{-29}\ \text{kg})(2.9979 \times 10^{8}\ \text{m/s})^2$

$= 2.798 \times 10^{-12}\ \text{kg m}^2/\text{s}^2 = 2.798 \times 10^{-12}\ \text{J/nucleus}$

$2.798 \times 10^{-12}\ \text{J/nucleus} \times \dfrac{1\ \text{MeV}}{1.602177 \times 10^{-13}\ \text{J}} = 17.5\ \text{MeV}$

136-27 Isotopes such as ^{26}Al (half-life: 7.2×10^5 years) are believed to have been present in our solar system as it formed, but have since decayed and are now called extinct nuclides.

(a) ^{26}Al decays by β^+ emission or electron capture. Write the equations for these two nuclear transformations.

(b) The earth was formed about 4.7×10^9 (4.7 billion) years ago. How old was the earth when 99.999999% of the ^{26}Al originally present had decayed?

Solution

(a) $^{26}_{13}\text{Al} \longrightarrow {}^{26}_{12}\text{Mg} + {}^{0}_{+1}\text{e}$, $^{26}_{13}\text{Al} + {}^{0}_{-1}\text{e} \longrightarrow {}^{26}_{12}\text{Mg}$; (b) Find the value of the decay constant of Al-26. The half-life is 7.2×10^5 y:

$\lambda = \dfrac{\ln 2}{7.2 \times 10^5\ \text{y}} = \dfrac{0.6931}{7.2 \times 10^5\ \text{y}} = 9.626 \times 10^{-7}\ \text{y}^{-1}$

$t = \dfrac{\left(\ln \dfrac{1.0000000}{0.000001}\right)}{9.626} \times 10^{-7}\ \text{y}^{-1} = 1.4 \times 10^{7}\ \text{y}$

= 14 million years

136-28 Write a balanced equation for each of the following nuclear reactions:

(a) bismuth-212 decays into polonium-212

(b) beryllium-8 and a positron are produced by the decay of an unstable nucleus

(c) neptunium-239 forms from the reaction of uranium-238 with a neutron and then spontaneously converts into plutonium-239

(d) strontium-90 decays into yttrium-90

Solution

(a) $^{212}_{83}\text{Bi} \longrightarrow {}^{212}_{84}\text{Po} + {}^{0}_{-1}\text{e}$; (b) $^{8}_{5}\text{B} \longrightarrow {}^{8}_{4}\text{Be} + {}^{0}_{-1}\text{e}$; (c) $^{238}_{92}\text{U} + {}^{1}_{0}\text{n} \longrightarrow {}^{239}_{93}\text{Np} + {}^{0}_{-1}\text{Np}$, $^{239}_{93}\text{Np} \longrightarrow {}^{239}_{94}\text{Pu} + {}^{0}_{-1}\text{e}$; (d) $^{90}_{38}\text{Sr} \longrightarrow {}^{90}_{39}\text{Y} + {}^{0}_{-1}\text{e}$

136-29 Write a balanced equation for each of the following nuclear reactions:

(a) mercury-180 decays into platinum-176

(b) zirconium-90 and an electron are produced by the decay of an unstable nucleus
(c) thorium-232 decays and produces an alpha particle and a radium-228 nucleus, which decays into actinium-228 by beta decay
(d) neon-19 decays into fluorine-19

Solution

(a) ${}^{27}_{12}\text{Mg} \longrightarrow {}^{27}_{13}\text{Al} + {}^{0}_{-1}\text{e}$; (b) ${}^{180}_{80}\text{Hg} \longrightarrow {}^{176}_{78}\text{Pt} + {}^{4}_{2}\alpha$; (c) ${}^{90}_{39}\text{Y} \longrightarrow {}^{90}_{40}\text{Zn} + {}^{0}_{-1}\text{e}$; (d) ${}^{232}_{90}\text{Th} \longrightarrow {}^{4}_{2}\alpha + {}^{228}_{88}\text{Ra}$, ${}^{228}_{88}\text{Ra} \longrightarrow {}^{228}_{89}\text{Ac} + {}^{0}_{-1}\text{e}$; (e) ${}^{19}_{10}\text{Ne} \longrightarrow {}^{19}_{9}\text{F} + {}^{0}_{1}\text{e}$

Unit 137 – Transmutation and nuclear energy

137-1 When atoms of aluminum-27 are bombarded with alpha particles, a neutron and an element are produced. Provide the balanced nuclear equation and indicate the element that is produced.

Solution:

$${}^{27}_{13}\text{Al} + {}^{4}_{2}\text{He} \longrightarrow {}^{30}_{15}\text{P} + {}^{1}_{0}n$$

The element produced is phosphorus-30.

137-2 What does the decay of tin-110 by electron capture yield? Write the balanced nuclear equation.

Solution:

$${}^{110}_{50}\text{Sn} + {}^{0}_{-1}\text{e} \longrightarrow {}^{110}_{49}\text{In}$$

137-3 When atoms of magnesium-24 are bombarded with alpha particles, a neutron and an element are produced. What is the element? Write the balanced nuclear equation

Solution:

$${}^{24}_{12}\text{Mg} + {}^{4}_{2}\text{He} \longrightarrow {}^{27}_{14}\text{Si} + {}^{1}_{0}n$$

137-4 Write the balanced nuclear equation for the production ofberkelium-244, made by the bombardment of Am-241 with alpha particles.

Solution

$${}^{241}_{95}\text{Am} + {}^{4}_{2}\text{He} \longrightarrow {}^{244}_{97}\text{Bk} + {}^{1}_{0}\text{n}$$

137-5 How does nuclear fission differ from nuclear fusion? Why are both of these processes exothermic?

Solution

Nuclear fission is the conversion of heavy nuclei into two or more lighter nuclei and other fragments. Fusion is the combination of two smaller nuclei into a heavier nucleus. In both cases, the mass of the products is less than the mass of the reactants and the mass lost is converted into energy—thus yielding an exothermic reaction.

137-6 Both fusion and fission are nuclear reactions. Why is a very high temperature required for fusion, but not for fission?

Solution

Two nuclei must collide for fusion to occur. High temperatures are required to give the nuclei enough kinetic energy to overcome the very strong repulsion resulting from their positive charges.

137-7 Cite the conditions necessary for a nuclear chain reaction to take place. Explain how it can be controlled to produce energy, but not produce an explosion.

Solution

The number of neutrons produced by fission must equal or exceed the number of neutrons absorbed by the nuclei that split plus the number lost to the surroundings. The amount of fissionable material must match or exceed the critical mass. Insertion of a neutron-absorbing material, called a *moderator*, which is encased in a metal tube, into the nuclear fuel can be used to control the reaction.

137-8 Describe the components of a nuclear reactor.

Solution

A nuclear reactor consists of the following:

1. A nuclear fuel. A fissionable isotope must be present in large enough quantities to sustain a controlled chain reaction. The radioactive isotope is contained in tubes called fuel rods.
2. A moderator. A moderator slows neutrons produced by nuclear reactions so that they can be absorbed by the fuel and cause additional nuclear reactions.
3. A coolant. The coolant carries heat from the fission reaction to an external boiler and turbine where it is transformed into electricity.
4. A control system. The control system consists of control rods placed between fuel rods to absorb neutrons and is used to adjust the number of neutrons and keep the rate of the chain reaction at a safe level.
5. A shield and containment system. The function of this component is to protect workers from radiation produced by the nuclear reactions and to withstand the high pressures resulting from high-temperature reactions.

137-9 In usual practice, both a moderator and control rods are necessary to operate a nuclear chain reaction safely for the purpose of energy production. Cite the function of each and explain why both are necessary.

Solution

A moderator slows down neutrons that travel too fast to cause fission. The neutrons must be slowed so that they will be absorbed by the fuel, such as U-235, and produce additional nuclear reactions. Moderators can consist of heavy water (D_2O), graphite, carbon dioxide, or ordinary water. Control rods absorb neutrons. Cadmium or boron-10 are often used to keep the rate of the chain reaction at a safe level by adjusting the number of neutrons that would otherwise strike the fuel rods, causing the production of still more neutrons.

137-10 Describe how the potential energy of uranium is converted into electrical energy in a nuclear power plant.
Solution
fission of uranium generates heat, which is carried to an external steam generator (boiler). The resulting steam turns a turbine that powers an electrical generator.

137-11 The mass of a hydrogen atom $({}^{1}_{1}\text{H})$ is 1.007825 amu; that of a tritium atom $({}^{3}_{1}\text{H})$ is 3.01605 amu; and that of an α particle is 4.00150 amu. How much energy in kilojoules per mole of ${}^{4}_{2}\text{He}$ produced is released by the following fusion reaction: ${}^{1}_{1}\text{H} + {}^{3}_{1}\text{H} \longrightarrow {}^{4}_{2}\text{He}$.
Solution
Calculate the mass change that occurs; then convert this mass to energy:
mass defect = mass ${}^{1}_{1}\text{H}$ + mass ${}^{3}_{1}\text{H}$ – mass ${}^{4}_{2}\text{He}$ = 1.007825 amu + 3.010605 amu – 4.00150 amu = 0.022375 amu. Convert atomic mass units to kilograms:
Mass defect = 0.022375 amu × $1.6605 \times 10^{-27} \frac{\text{kg}}{\text{amu}}$ = 3.7154×10^{-29} kg
$E = mc^2 = (3.7154 \times 10^{-29}\ \text{kg})(2.9979 \times 10^8\ \text{m/s})^2 = 3.339 \times 10^{-12}\ \text{kg m}^2/\text{s}^2 = 3.339 \times 10^{-12}$ J/nucleus; on a per-mole basis:

$$E = \frac{E}{\text{nucleus}} \times \frac{6.022 \times 10^{23}}{\text{mol}} = 3.339 \times 10^{-12}\ \text{J/nucleus} \times 6.022 \times 10^{23} \frac{\text{nuclei}}{\text{mol}}$$
$$= 2.011 \times 10^{12}\ \text{J mol}^{-1}$$
$$= 2.011 \times 10^{9}\ \text{kJ mol}^{-1}$$

Unit 138 – Uses of Radioisotopes

138-1 How can a radioactive nuclide be used to show that the equilibrium:
$\text{AgCl}(s) \rightleftharpoons \text{Ag}^+(aq) + \text{Cl}^-(aq)$
is a dynamic equilibrium?
Solution
Introduction of either radioactive Ag^+ or radioactive Cl^- into the solution containing the stated reaction, with subsequent time given for equilibration, will produce a radioactive precipitate that was originally devoid of radiation.

138-2 Technetium-99m has a half-life of 6.01 hours. If a patient injected with technetium-99m is safe to leave the hospital once 75% of the dose has decayed, when is the patient allowed to leave?
Solution
75% decayed is equivalent to 25% remaining. This represents two half-lives $\left(\frac{1}{2}\right)^2 = 0.25$, so the patient would be free to go after 12.02 hours

138-3 Iodine that enters the body is stored in the thyroid gland from which it is released to control growth and metabolism. The thyroid can be imaged if iodine-131 is injected into the body. In larger doses, I-133 is also used as a means of treating cancer of the thyroid. I-131 has a half-life of 8.70 days and decays by β^- emission.

(a) Write an equation for the decay.

(b) How long will it take for 95.0% of a dose of I-131 to decay?

Solution

(a) ${}^{133}_{53}\text{I} \longrightarrow {}^{133}_{54}\text{Xe} + {}^{0}_{-1}\text{e}$;

(b) First, find the value of λ:

$$\lambda = \frac{0.6931}{8.70\text{ day}} = 0.07967\text{ day}^{-1}$$

$$\frac{\ln c_0}{c} = \lambda t;\ \ln\left(\frac{1.000}{0.050}\right) = 0.07967\text{ day}^{-1}$$

$$t = \frac{2.996}{0.07967\text{ day}^{-1}} = 37.6\text{ days}$$

Unit 139 – Biological Effects of Radiation

139-1 If a hospital were storing radioisotopes, what is the minimum containment needed to protect against:

(a) cobalt-60 (a strong γ emitter used for irradiation)

(b) molybdenum-99 (a beta emitter used to produce technetium-99 for imaging)

Solution

(a) thick shielding made of lead; (b) thin metal shielding

139-2 Based on what is known about Radon-222's primary decay method, why is inhalation so dangerous?

Solution

Alpha particles can be stopped by very thin shielding but have much stronger ionizing potential than beta particles, X-rays, and γ-rays. When inhaled, there is no protective skin covering the cells of the lungs, making it possible to damage the DNA in those cells and cause cancer.

139-3 Given specimens uranium-232 ($t_{1/2}$ = 68.9 y) and uranium-233 ($t_{1/2}$ = 159,200 y) of equal mass, which one would have greater activity and why?

Solution

The sample of uranium-232 would have greater activity because half-life is inversely related to activity, so the material with the shorter half-life would have the larger activity if the atomic masses are similar.

139-4 A scientist is studying a 2.234 g sample of thorium-229 ($t_{1/2}$ = 7340 y) in a laboratory.

(a) What is its activity in Bq?

(b) What is its activity in Ci?

Solution

$$\text{Activity} = \lambda N = \left(\frac{\ln 2}{t_{1/2}}\right)N = \left(\frac{\ln 2}{7340 \text{ y}}\right)2.234 \text{ g} = 9.162 \times 10^{-5} \frac{\text{g}}{\text{y}}$$

(a) Converted to Bq:

$$9.162 \times 10^{-5} \frac{\text{g}}{\text{y}} \times \frac{1 \text{ y}}{365 \text{ d}} \times \frac{1 \text{ d}}{24 \text{ h}} \times \frac{1 \text{ h}}{3600 \text{ s}} \times \frac{1 \text{ mol}}{229 \text{ g}} \times \frac{6.02 \times 10^{23} \text{ atoms}}{1 \text{ mol}} \times \frac{1 \text{ decay}}{1 \text{ atom}}$$

$$= 7.64 \times 10^{9} \frac{\text{decays}}{\text{s}} = 7.64 \times 10^{9} \text{ Bq}$$

(b) Converted to Ci:

$$7.64 \times 10^{9} \frac{\text{decays}}{\text{s}} \times \left(\frac{1 \text{ Ci}}{3.7 \times 10^{11} \frac{\text{decays}}{\text{s}}}\right) = 2.06 \times 10^{-2} \text{ Ci}$$

139-5 Given specimens neon-24 ($t_{1/2}$ = 3.38 min) and bismuth-211 ($t_{1/2}$ = 2.14 min) of equal mass, which one would have greater activity and why?

Solution

Because the atomic masses are dissimilar, calculations are needed to determine activity.

$$\text{Activity Ne} = \lambda N = \left(\frac{\ln 2}{t_{1/2}}\right)N = \left(\frac{\ln 2}{3.38 \text{ min}}\right)1.000 \text{ g} = 0.089 \frac{\text{g}}{\text{min}}$$

$$\text{Activity Bi} = \lambda N = \left(\frac{\ln 2}{t_{1/2}}\right)N = \left(\frac{\ln 2}{2.14 \text{ min}}\right)1.000 \text{ g} = 0.14 \frac{\text{g}}{\text{min}}$$

While this appears to favor bismuth-211, the difference in masses will determine the absolute rate of decays occurring:

$$\text{Ne: } 0.089 \frac{\text{g}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{1 \text{ mol}}{24 \text{ g}} \times \frac{6.02 \times 10^{23} \text{ atoms}}{1 \text{ mol}} \times \frac{1 \text{ decay}}{1 \text{ atom}} = 3.7 \times 10^{19} \frac{\text{decays}}{\text{s}}$$

$$\text{Bi: } 0.14 \frac{\text{g}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{1 \text{ mol}}{211 \text{ g}} \times \frac{6.02 \times 10^{23} \text{ atoms}}{1 \text{ mol}} \times \frac{1 \text{ decay}}{1 \text{ atom}} = 6.6 \times 10^{17} \frac{\text{decays}}{\text{s}}$$

Thus, due to the smaller atomic mass, the neon-24 sample will have higher activity, despite having a longer half-life than the bismuth-211 sample.